

Higher Dimensional Operators or Large Extra Dimensions?

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Abstract

We deform gravity with higher curvature terms in four dimensions and argue that the non-relativistic limit is of the same form as the non-relativistic limit of the theories with large extra dimensions. Therefore the experiments that perform sub-millimeter tests of inverse-square law cannot distinguish the effects of large extra dimensions from the effects of higher dimensional operators. In other words instead of detecting the presence of sub-millimeter dimensions; the experiments could be detecting the existence of massive modes of gravity with large masses ($\geq 10^{-3}$ eV).

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In the context of “brane world” pictures modification of gravity both at small and large distances has been a subject of recent discussions [1, 2, 3, 4, 5, 6]. During these discussions an old puzzle, about the mass of the gravitons re-emerged: van Dam-Veltman-Zakharov discontinuity.

In the papers [7, 8] it was argued that predictions of the gravity theory with strictly massless gravitons and the theory with arbitrarily small massive gravitons are different. The story is well-summarized in a recent paper [9] but we shall briefly recapture what the issue is before we address our problem.

The point raised in [7, 8] for massive gravity is rather puzzling since it seems that massless limit of massive gravity is not General Relativity. In fact starting with the Einstein-Hilbert action

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} R, \quad (1)$$

in the weak field limit one obtains the usual Newton’s potential

$$V(r) = -\frac{GM}{r} \quad (2)$$

On the other hand if the action is augmented by a Pauli-Fierz mass term ²

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} \left\{ R + \frac{m_g^2}{4} [h_{\mu\nu}^2 - (h_\mu^\nu)^2] \right\} \quad (3)$$

where $h_{\mu\nu}$ is a small perturbation around the Minkowski space, one obtains the potential

$$V(r) = -\frac{4}{3} \frac{GM}{r} e^{-m_g r} \quad (4)$$

Therefore in the limit of vanishing graviton mass the two potentials do not match. One can cure the puzzle by redefining the Newton’s constant of the latter theory but the prediction of massive gravity for the bending of light around the Sun would be off by 25%. At this level of perturbation theory there seems to be no room for a massive graviton no matter how small mass it has.

²Boulware and Deser [10] argue that there is no consistent quantum theory of massive spin-2 fields in four dimensions.

Resolution of this puzzle was given by Vainshtein [11, 9] who showed that one needs to be careful in taking the small m_g limit. Starting from the Schwarzschild solution in the massive case he showed that $m_g \rightarrow 0$ limit is singular and a proper limiting procedure shows that there is in fact no discontinuity. Another resolution of this puzzle was given in the recent works [12, 13] who showed that there is no discontinuity in massless graviton limit if there is a cosmological constant of both signs. See also the earlier work [14] on dS_4 . It was reported in [15] that there is no discontinuity for generic backgrounds with non-constant curvature invariants.

Even after one resolves van-Dam-Veltman-Zakharov puzzle ‘a la Vainshtein, clearly the mass one can give to the graviton through Pauli-Fierz term should be tiny. Experimental astrophysical limits allow m_g to be at most $(10^{25}cm)^{-1}$ [16].

There is an other way to study the modifications of gravity in four dimensions; namely by adding higher curvature terms in the action. In this Letter we start with the following action,

$$S = \int d^4x \sqrt{-g} \left\{ \frac{R}{16\pi G} + \alpha R^2 + \beta R_{\mu\nu} R^{\mu\nu} + \gamma R_{\mu\nu\lambda\delta} R^{\mu\nu\lambda\delta} \right\} \quad (5)$$

We shall try to answer the following questions. What are the experimentally allowed values of α , β and γ ? Do they have to be strictly zero? If not zero how large can they be? We will find that the answers to these questions seriously effect the interpretations of the sub-millimeter experiments on gravity which try to “see” large extra dimensions.

Observe that at this stage if one sets

$$\beta = -4\alpha \quad \gamma = \alpha \quad (6)$$

Then one obtains Einstein-Hilbert-Gauss-Bonnet action

$$S = \int d^4x \sqrt{-g} \left\{ \frac{R}{16\pi G} + \alpha(R^2 - 4R_{\mu\nu} R^{\mu\nu} + R_{\mu\nu\lambda\delta} R^{\mu\nu\lambda\delta}) \right\} \quad (7)$$

This action appears in the string-generated models of gravity [17, 18]. The last three terms combine to make a topological invariant $\epsilon^{\mu\nu\rho\sigma} \epsilon_{\alpha\beta\gamma\delta} R_{\mu\nu}{}^{\alpha\beta} R_{\rho\sigma}{}^{\gamma\delta}$ and therefore do not effect

the equations of motion in four dimensions. In particular Schwarzschild solution remains intact and the Newton's potential outside the horizon of a massive spinless particle is

$$V(r) = -\frac{GM}{r} \quad (8)$$

Clearly it is not possible to detect/measure the existence of the topological term and α can take any, in particular large, value. This Gauss-Bonnet limit (6) is not particularly interesting as far as classical dynamics is concerned.

Now we shall take a different route and start with generic values of α , β and γ and compute the observable effects of these dimensionless parameters.

We shall consider the scattering of two massive spin-0 particles in the non-relativistic limit.

$$S_M = \frac{1}{2} \int d^4x \sqrt{-g} \{g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - M^2 \phi^2\} \quad (9)$$

With no effort one can see that the action (5) can be written in the following form

$$S = \int d^4x \sqrt{-g} \left\{ \frac{R}{16\pi G} + \tilde{\alpha} R^2 + \tilde{\beta} R_{\mu\nu} R^{\mu\nu} \right\} + S_{GB} \quad (10)$$

where the Gauss-Bonnet action is

$$S_{GB} = \gamma \int d^4x \sqrt{-g} \{R^2 - 4R_{\mu\nu} R^{\mu\nu} + R_{\mu\nu\lambda\delta} R^{\mu\nu\lambda\delta}\}, \quad (11)$$

given that one performs the following identification

$$\tilde{\alpha} = \alpha - \gamma \quad \tilde{\beta} = \beta + 4\gamma \quad (12)$$

The theory defined by the action (apart from the topological term) (10) has been studied since the early days of the emergence of Einstein's theory of gravity. [20, 21, 22, 23]. We can borrow the non-relativistic potential from [20, 23]

$$V(r) = GM \left\{ -\frac{1}{r} + \frac{4}{3} \frac{e^{-m_1 r}}{r} - \frac{1}{3} \frac{e^{-m_0 r}}{r} \right\} \quad (13)$$

where ³

$$m_0^2 = \frac{1}{(3\tilde{\alpha} + \tilde{\beta})32\pi G} \quad m_1^2 = -\frac{1}{\tilde{\beta}16\pi G} \quad (14)$$

³One thing to notice is that for finite masses the potential for large distances reduces to the usual Newtonian limit and near the origin it is finite and we have $V(r) = \frac{GM}{3} \{(m_0 - 4m_1) - (4m_1^2 - m_0^2)\frac{r}{2} + O(r^2)\}$

There are three modes in the theory one with a vanishing mass which gives the Newton force and the two massive modes which create Yukawa-type interactions.

One needs to impose the condition that there are no tachyons in the theory which yields ⁴

$$3\tilde{\alpha} + \tilde{\beta} > 0 \qquad -\tilde{\beta} > 0 \qquad (15)$$

We have gotten rid off the tachyons but the second term in the potential (13) has a wrong sign. Ghosts are expected for generic values of $\tilde{\alpha}$ and $\tilde{\beta}$. ⁵ Fortunately we can also get rid of the ghost by choosing $\tilde{\beta} \sim 0$. m_1 becomes infinitely heavy and decouples from the theory. One then has the following potential ⁶

$$V(r) = -GM \left\{ \frac{1}{r} + \frac{1}{3} \frac{e^{-m_0 r}}{r} \right\} \qquad (16)$$

Now we have

$$m_0^2 = \frac{1}{\tilde{\alpha} 96 \pi G} \qquad (17)$$

Recent experiments in gravity [24] tested $1/r^2$ law down to sub-millimeter ($0.1mm$). One can safely take m_0 of the order of, or bigger than, $10/mm$ with no observable effects as far as gravitational potential is concerned. We still need to check whether the bending of light by the Sun's gravitational field predicted by the higher curvature theory (10) is

⁴The theory which results when one from the onset sets $3\tilde{\alpha} = -\tilde{\beta}$ is known as Einstein-Bach-Weyl gravity and it was studied in [20]. Incidentally there seems to be a discontinuity in the spirit of van-Dam-Veltman and Zakharov; namely the non-relativistic limit and the limit of $3\tilde{\alpha} = -\tilde{\beta}$ do not seem to commute. We expect this puzzle will be resolved 'a la Vainshtein but we shall not discuss it here.

⁵I am grateful for S. Deser for stressing this point. I. Kogan pointed out to me that negative norm states ('radions') appear also in the brane world theories with negative tension branes [See the discussion and references in [19]].

⁶An other interesting theory is when $m_0 = m_1$ which can be obtained by setting $2\tilde{\alpha} = -\tilde{\beta}$ the non-relativistic potential reads $V(r) = GM \left\{ -\frac{1}{r} + \frac{e^{-m_0 r}}{r} \right\}$ In the limit of large $\tilde{\alpha}$ the mass goes to zero ($m_0 \rightarrow 0$) and the potential vanishes. The theory in a sense becomes topological and the propagating graviton disappears. The Lagrangian reduces to $\mathcal{L} = \tilde{\alpha} \{ R^2 - 2R_{\mu\nu}R^{\mu\nu} \}$, or in terms of the starting action (5) whenever $2\alpha + \beta + 6\gamma = 0$ is satisfied one gets a theory with no Newtonian limit for large α . Needless to say that this theory is not compatible with the experiments. The massive modes should in fact be quite heavy to escape large violations of Newton's law in large distances.

consistent with the experiments or not. This computation was carried out in [22]. Below I shall summarize their result. Around the Sun the metric reads

$$ds^2 = g_{00}dt^2 - f(r)d\vec{x}^2 \quad (18)$$

where

$$\begin{aligned} g_{00} &= 1 + \frac{2MG}{r} \left\{ -1 - \frac{1}{3}e^{-m_0 r} + \frac{4}{3}e^{-m_1 r} \right\} \\ f(r) &= 1 - \frac{2MG}{r} \left\{ -1 + \frac{1}{3}e^{-m_0 r} + \frac{2}{3}e^{-m_1 r} \right\} \end{aligned} \quad (19)$$

Then for the null geodesic $ds^2 = 0$ the index of refraction reads

$$n(r) = \sqrt{\frac{f}{g_{00}}} = 1 + \frac{2MG}{r} - \frac{2MG}{r}e^{-m_1 r} + O(G^2) \quad (20)$$

Crucial observation is that the massive scalar mode (m_0) do not contribute to the deflection of light. The contribution of the ghost state is exponentially suppressed and goes to zero for infinite (m_1) which is the limit we consider here.

To conclude in this letter we have studied the various limits of higher curvature gravity in four dimensions. Higher curvature terms generate massive modes in addition to the usual massless mode in gravity. In particular the Lagrangian

$$\mathcal{L} = \frac{R}{16\pi G} + \tilde{\alpha}R^2 \quad (21)$$

has a massless and a massive mode whose mass can be $\geq 10^{-3}\text{eV}$ without contradicting the experimental tests of gravity: the $1/r^2$ law and the bending of light by the Sun.

We need to stress a crucial point with regard to the experiments on large extra dimensions. Theories with large extra dimensions predict non-relativistic potential of the form [1, 3, 25]

$$V(r) = -GM \left\{ \frac{1}{r} + a \frac{e^{-br}}{r} \right\} \quad (22)$$

Clearly this potential is of the form one obtains in higher curvature gravity (16) if a is of the order of unity. In fact starting from higher dimensions and compactifying on spheres

or tori a turns out to be $O(1)$ or for Calabi-Yau compactifications a can be at most as big as 20 [25]. Therefore if a is not too large ⁷ the non-relativistic potentials derived from large extra dimensions will coincide with the potential derived from higher curvature gravity. It would be hard to decide whether the experimentalists are measuring the effects of large extra dimensions or the effects of massive modes in higher curvature gravity; namely the effects of higher dimensional operators with large numerical coefficients.

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⁷See [26] for the possibility of large a in certain supersymmetric theories when moduli fields and the dilaton are taken into account

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